

HOMEWORK 4
PSTAT 120A: Summer 2022

Due: 11:59pm on Tuesday, July 5
Instructor: Ethan P. Marzban

Instructions:

- Please submit your work to Gradescope by no later than the due date posted above.
- Be sure to show your work; correct answers with no supporting work will not be awarded full points.
- 2 randomly selected questions/parts will be graded, but you must still turn in your work for all problems in order to be eligible to earn full credit.

1. Consider a random variable X with probability density function (p.d.f.) given by

$$f_X(x) = \begin{cases} \frac{c}{x^3} & \text{if } x \geq 2 \\ 0 & \text{otherwise} \end{cases}$$

where $c > 0$ is an as-of-yet undetermined constant.

- Find the value of c that ensures the function $f_X(x)$ is a valid p.d.f.
 - Compute $P(X \geq 5 \mid X \geq 3)$.
 - Compute $E[X]$
 - Compute $\text{Var}(X)$
 - Find $F_X(x)$, the cumulative distribution function (c.d.f.) of X .
 - Find the 72nd percentile of the distribution of X .
2. A company manufactures steel rods that are to be of length 15 meters. However, due to imperfections in the manufacturing process, the length of a rod is actually uniformly distributed between 13 meters and 17 meters.
- What is the probability that a randomly selected rod will be longer than 16 meters?
 - A sample of 100 rods is taken with replacement, and the number of rods that are longer than 16 meters is recorded. What is the probability that this sample of 100 rods contains at least 63 rods that are longer than 16 meters? **You may leave your answer as an unsimplified finite sum.**
3. On a particular stretch of Highway 101, the speed limit is listed as 65mph. In actuality, the speed of a randomly selected car is a random variable that follows the normal distribution with mean 60mph and standard deviation 4mph.
- What is the probability that a randomly selected car will be speeding? (Here, speeding just means "travelling at a speed greater than the speed limit") **Leave your answer in terms of $\Phi(\cdot)$, the standard normal c.d.f..**
 - Samantha's car is travelling at the 75th percentile of speeds. How fast is Samantha travelling? **Leave your answers in terms of $\Phi^{-1}(\cdot)$, the inverse of the standard normal c.d.f..**
4. Cars arrive at a traffic light according to a Poisson Point Process at a constant rate of 12 cars per hour.
- What is the probability that 10 cars arrive at the traffic light in a given one-hour interval?
 - What is the probability that 5 cars arrive at the traffic light in a given 30-minute interval?
 - What is the probability that the 4th and 5th cars arrive within 5 minutes of each other?
 - What is the expected amount of time (in hours) that we must wait between the arrival of the 1st and 4th cars?