HOMEWORK 6

PSTAT 120A: Summer 2022

Due: 11:59pm on Friday, July 15

Instructor: Ethan P. Marzban

Instructions:

- Please submit your work to Gradescope by no later than the due date posted above.
- Be sure to show your work; correct answers with no supporting work will not be awarded full points.
- 2 randomly selected questions/parts will be graded, but you must still turn in your work for all problems in order to be eligible to earn full credit.

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1. Let (X, Y) be a random vector with joint p.d.f. given by

$$f_{X,Y}(x,y) = \begin{cases} c \cdot \left(\frac{y}{x} \right)^4 & \text{if } (x,y) \in \mathcal{R} \\ 0 & \text{otherwise} \end{cases}$$

where $c > 0$ is an as-of-yet undetermined constant, and $\mathcal{R}$ is the region in the first quadrant below the graph of $y = \min\{x, 1\}$.

   (a) Find the value of c.
   (b) Set up, but do not evaluate, the double integral corresponding to $\mathbb{P}(X + Y \geq 2)$.
   (c) Find $f_X(x)$, the marginal p.d.f. of X.
   (d) Find $f_Y(y)$, the marginal p.d.f. of Y.
   (e) Find $\mathbb{E}[X]$
   (f) Find $\mathbb{E}[Y]$
   (g) Compute $\mathrm{Cov}(X, Y)$.
   (h) Are X and Y independent? Explain.

2. Let $X_i \sim \mathrm{Exp}(\lambda_i)$ for $i = 1, \cdots, n$; further suppose that the X_i's are independnt. Define

$$Y := \min_{1 \leq i \leq n} \{X_i\}$$

in other words, Y is the smallest of the X_i's. Identify the distribution of Y **by name**; be sure to include any/all relevant parameter(s)!

3. Let (X, Y) be a random vector with joint p.m.f. given by

$$p_{X,Y}(x,y) = \begin{cases} (y-1) \left(\frac{1}{2} \right)^{x+y} & \text{if } x \in \{1, 2, \cdots\},\ y \in \{2, 3, \cdots\} \\ 0 & \text{otherwise} \end{cases}$$

   (a) Verify that $p_{X,Y}(x,y)$ is a valid joint p.m.f.
   (b) Find the marginal p.m.f.'s of X and Y. **Hint:** You can answer this question without doing any additional summations!
   (c) Use your answer to part (b) to compute $\mathbb{E}[X]$ and $\mathbb{E}[Y]$. **Hint:** You can answer this question without doing any additional summations!

(d) Compute $\mathbb{E}[XY]$. **Hint:** You can answer this question without doing any additional summations! Just be sure to justify all of your work/steps.

4. Clara and Donna both roll a fair $k-$sided die, independently of each other.

   a) Compute the probability that Clara rolls a number smaller than Donna.

   b) Consider the probability that Clara rolls a number strictly greater than Donna. A student argues that this probability should simply be 1 minus the answer to part (a). Do you agree with this student's reasoning? Explain why or why not.