

**HOMEWORK 9**  
PSTAT 120A: Summer 2022

**Due: 11:59pm on Tuesday, July 26, 2022**  
Instructor: Ethan P. Marzban

**Instructions:**

- Please submit your work to Gradescope by no later than the due date posted above.
- Sometime on Tuesday morning, the solutions to this homework will become available (yes, before the due date).
- Be sure to show your work; correct answers with no supporting work will not be awarded full points.
- 2 randomly selected questions/parts will be graded, but you must still turn in your work for all problems in order to be eligible to earn full credit.

1. The hit new restaurant *Gaucha Gourmande* is trying to set their menu prices.

It is known that the true cost of making their signature dish varies from day to day (due to changes in market pricing of ingredients). We assume that the cost of manufacturing this dish on any given day is a random variable with mean \$30 and standard deviation \$2, independent of all other days.

The owners of *Gaucha Gourmande* would like to set the price of the dish to be a fixed number \$ $m$ , and would like to ensure that with probability 0.99 the revenue generated from selling the dish exceeds the true cost of manufacturing it in a 100-day period. [Assume, somewhat unrealistically, that the restaurant sells only one dish per day.]

- (a) Find the value of  $m$  using Chebyshev's Inequality
- (b) Find the value of  $m$  using the Central Limit Theorem

2. After Session A ends, Ethan is a bit low on funds. As such, he decides to go back to basics and start up a lemonade stand! He's not very good, though (customer reviews seem to be rife with complaints about how he manages to somehow turn everyday scenarios into math problems?); on average, he only makes about \$10 a day. His true earnings on any given day is, however, a random variable. Additionally, Ethan finds that his earnings are correlated across days; intuitively, it makes sense that earnings on days that are farther apart would be less correlated, so the following covariance structure is adopted:

$$\text{Cov}(X_i, X_j) = \left(\frac{1}{2}\right)^k \text{ for } |i - j| = k, \quad i, j \in \{1, 2, \dots\}$$

Ethan is interested in his net (total) earnings over a 30-day period.

- (a) What is the expected total earnings Ethan will make over these 30 days?
- (b) What is the variance of the total earnings Ethan will make over these 30 days?
- (c) Approximate the probability that Ethan will earn more than \$310 over these 30 days.

3. Let  $(X, Y)$  be a pair of bivariate random variables with joint probability density function (p.d.f.) given by

$$f_{X,Y}(x, y) = \begin{cases} \frac{\lambda}{y} \cdot e^{-\lambda y} & \text{if } 0 < x < y < \infty \\ 0 & \text{otherwise} \end{cases}$$

where  $\lambda > 0$  is a fixed, deterministic constant.

- (a) Show that  $f_{X,Y}(x,y)$  is a valid probability density function.
- (b) Identify the marginal distribution of  $Y$ .
- (c) Identify the conditional distribution of  $(X | Y)$ .
- (d) Using your answer to the above parts of this question, and **without performing any integration**, compute  $\mathbb{E}[X]$ .