PSTAT 120A: Rough Overview of Course Topics

1 Axioms of Probability; Counting

Corresponding Textbook Sections: §1.1 - 1.4; Appendices B and C

- Experiment; outcome space, event space
- Probability measures; axioms of probability
- Probability Space
- Fundamental Principle of Counting
- Slot Diagram/Slot Method
- $(n)_{k}, \binom{n}{k}$
- Basic Set Theory: union, intersection, complement, set difference, Cartesian Product, cardinality; disjoint; DeMorgan's Laws; partitions
- Classical Definition of Probability (Equally Likely Outcomes)

2 Conditional Probability and Independence

Corresponding Textbook Sections: §2.1 - 2.3, 2.5

- Definition and interpretation of conditional probability
- Definition and interpretation of independence of two events
- Mutual independence; pairwise independence; conditional independence
- Law of Total Probability
- Bayes' Theorem/Rule

3 Random Variables

Corresponding Textbook Sections: §1.5; 3.1 - 3.4

- Definition of a random variable
- State Space
- Classification of random variables (discrete vs. continuous)

4 Distributions (Discrete and Continuous)

Corresponding Textbook Sections: §2.4; Chapter 3; 4.4 - 4.6; 5.2

- Conceptual definition of a distribution
- Probability Mass Function (p.m.f.); Probability Density Function (p.d.f.)
- Cumulative Mass Function (c.m.f.); Cumulative Distribution Function (c.d.f.)
- Expectation; moments; variance
- Binomial Distribution; Geometric Distribution; Negative Binomial Distribution; Discrete Uniform Distribution; Hypergeometric Distribution; Poisson Distribution
- Continuous Random Variables
- Named continuous distributions (Normal, Exponential, Uniform, Gamma)
- Poisson Point Processes
- Transformations of Random Variables (Change of Variable; CDF method)

5 Multivariate Probability

Corresponding Textbook Sections: Chapter 6; §7.1; 5.1; 8.1 - 8.4

- Double Integrals
- Random vectors
- Joint probability density/mass functions; construction, and utilization (probabilities, LO-TUS, etc.)
- Independence of Random Variables
- Covariance and Correlation
- Expectation and Variance of Linear Combinations of Random Variables
- Distribution of sums of random variables (convolution)
- Indicators and Expectation/Variance
- Moment-Generating Functions

6 Tail Bounds and Limit Theorems

Corresponding Textbook Sections: Chapter 9

- Markov's Inequality
- Chebyshev's Inequality
- Law of Large Numbers
- Central Limit Theorem
- DeMoivre-Laplace Theorem

7 Conditional Distributions

Corresponding Textbook Sections: §10.1 - 10.3

- Conditional Distributions
- Conditional probability density/mass functions
- Conditional Expectations
- Law of Iterated Expectations (a.k.a. Tower Property)
- Law of Iterated Variances