Score: ___________ / **35**

PSTAT 120A / **MIDTERM EXAM** / Summer 2022 Instructor: **Ethan Marzban**

NAME: ______________________________________ **Perm Number:** _____________________

SECTION (circle one): **3:30 - 4:20pm (Lucas)** **5 - 5:50pm (Moya)** **8 - 8:50am (Moya)**

Instructions:

- You will have **55 minutes** to complete this exam.
- You are allowed the use of a single **8.5 × 11-inch** sheet, front and back, of notes. You are also permitted the use of **calculators**; the use of any and all other electronic devices (laptops, cell phones, etc.) is prohibited.
- Unless otherwise specified, simplification is not needed; however, all integrals and infinite sums (unless otherwise specified) must be evaluated.
  - One exception is that, whenever applicable, answers may be left in terms of Φ, the standard normal c.d.f..
- **Problem 9(c) is a bonus question; please note that bonus questions will be graded on an all-or-nothing scale.**
- Good Luck!!!

Honor Code: In signing my name below, I certify that all work appearing on this exam is entirely my own and not copied from any external source. I further certify that I have not received any unauthorized aid while taking this exam.

× ____

Multiple Choice Questions:

Question:	1	2	3	4	5	Total
Points:	1	1	1	1	1	5
Score:						

Short-Answer Questions:

Question:	6	7	8	9	Total
Points:	5	11	8	6	30
Score:					

1 Multiple Choice Questions

Please fill in the bubble(s) ***on the exam below*** *corresponding to your answer. You do not need to submit any additional work for these questions.*

1. Which of the following statements is true in general? [1pts.]

   - ◯ Pairwise independence implies mutual independence.
   - ◯ Two mutually dependent events can be conditionally independent
   - ◯ There are 2^n computations needed to establish the mutual independence of n events
   - ◯ Pairwise independence is a stronger condition than mutual independence.
   - ◯ All of the above answer choices are **false**.

2. Given a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ and three events $A, B, C \in \mathcal{F}$, which of the following correctly computes $\mathbb{P}(A_1 \cup A_2 \cup A_3)$? [1pts.]

   - ◯ $\mathbb{P}(A_1) + \mathbb{P}(A_2) + \mathbb{P}(A_3)$
   - ◯ $1 - \mathbb{P}(A_1^{\complement}) \cdot \mathbb{P}(A_2^{\complement} \mid A_1^{\complement}) \cdot \mathbb{P}(A_3^{\complement} \mid A_1^{\complement} \cap A_2^{\complement})$
   - ◯ $\mathbb{P}(A_1) \cdot \mathbb{P}(A_2) \cdot \mathbb{P}(A_3)$
   - ◯ $1 - \mathbb{P}(A_1) - \mathbb{P}(A_2) - \mathbb{P}(A_3)$
   - ◯ None of the other answer choices

3. **Fill in the Blanks:** Discrete random variables have state spaces that are __________, whereas continuous random variables have state spaces that are __________. [1pts.]

   - ◯ finite; infinite
   - ◯ countable; uncountable
   - ◯ at most countable; uncountable
   - ◯ uncountable; at most countable
   - ◯ uncountable; countable
   - ◯ None of the above.

4. Consider a random variable X with p.m.f. given by [1pts.]

k	-1	2
$p_X(k)$	$1/4$	$3/4$

Which of the following is the correct value of $\mathbb{E}[X]$?

- ◯ 0
- ◯ $1/2$
- ◯ $3/4$
- ◯ 1
- ◯ $5/4$
- ◯ None of the above

5. In a bag of 100 marbles, 40 are blue and the remaining 60 are gold. Yaz draws marbles one by one at random, <u>replacing</u> the marble each time. If X denotes the number of marbles (including the final marble) Yaz has to draw before she observes her 3rd blue marble, which of the following accurately describes the distribution of X? [1pts.]

- ◯ $\text{Bern}(40)$
- ◯ $\text{Bern}(0.4)$
- ◯ $\text{Bin}(3, 0.4)$
- ◯ $\text{NegBin}(40, 0.4)$
- ◯ $\text{NegBin}(3, 0.4)$
- ◯ $\text{HyperGeom}(40, 100, 3)$
- ◯ $\text{Poisson}(0.4)$
- ◯ None of the above.

2 Short Answer Questions

Please mark your final answers in the spaces provided below each question. ***Be sure to show all of your work!***

6. Consider a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ and suppose A and B are two events. Prove the identity [5pts.]

$$\mathbb{P}(A \setminus B) = \mathbb{P}(A) \cdot \mathbb{P}(B^{\complement} \mid A)$$

7. Let X be a continuous random variable with probability density function (p.d.f.) given by

$$f_X(x) = \begin{cases} \frac{2}{25} \cdot x & \text{if } 0 \leq x \leq 5 \\ 0 & \text{otherwise} \end{cases}$$

(a) Verify that $f_X(x)$ is a valid probability density function. [3pts.]

(b) Compute [4pts.]

$$\mathbb{E}\left[\frac{1}{(1+X^2)}\right]$$

Show all of your steps, including any integration you perform!

(c) Find $F_X(x)$, the cumulative distribution function (c.d.f.) of X. Be sure to consider all cases! [4pts.]

8. The swanky new *GauchoStay* hotel is under construction! But, things are a bit behind schedule; there is currently only a 15% chance that a randomly selected room will have a fridge in it, independently of all other rooms. A contractor goes from room to room, examining which rooms have fridges and which do not, however they are a bit forgetful and could visit the same room twice. **For this problem, there is no need to simplify your answers.**

   (a) What is the probability that the contractor observes exactly 4 rooms with fridges among the first 10 rooms they examine? [2pts.]

   (b) What is the probability that the 13^{th} room the contractor examines is the fourth room with a fridge they observe? [2pts.]

(c) What is the expected number of rooms the contractor must visit before observing their third room with a fridge? [2pts.]

(d) Now, suppose that there are 200 rooms at *GauchoStay* and 30 of them have fridges. Additionally, suppose that the contractor now takes care to not examine the same room twice. What is the probability that the contractor observes exactly 5 rooms with fridges in a sample of 12 rooms they examine? [2pts.]

9. In a drawer, you have 2 red socks, 2 white socks, and 2 green socks. You randomly draw a sample of 4 socks, without replacement; let X denote the number of matching pairs in your sample (by matching, we mean in color).

   (a) What is the state space S_X of X? [2pts.]

   (b) Find the probability mass function of X. [4pts.]

(c) Compute $\mathbb{E}[X]$. [1 (bonus)]

You may use this page for scratch work, if necessary.

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