

NAME: \_\_\_\_\_

Perm Number: \_\_\_\_\_

SECTION (circle one):    3:30 - 4:20pm (Lucas)    5 - 5:50pm (Moya)    8 - 8:50am (Moya)

**Instructions:**

- You will have **55 minutes** to complete this exam.
- You are allowed the use of a single **8.5 × 11-inch** sheet, front and back, of notes. You are also permitted the use of **calculators**; the use of any and all other electronic devices (laptops, cell phones, etc.) is prohibited.
- Unless otherwise specified, simplification is not needed; however, all integrals and infinite sums (unless otherwise specified) must be evaluated.
  - One exception is that, whenever applicable, answers may be left in terms of  $\Phi$ , the standard normal c.d.f..
- **Problem 9(c) is a bonus question; please note that bonus questions will be graded on an all-or-nothing scale.**
- Good Luck!!!

**Honor Code:** In signing my name below, I certify that all work appearing on this exam is entirely my own and not copied from any external source. I further certify that I have not received any unauthorized aid while taking this exam.

× \_\_\_\_\_

**Multiple Choice Questions:**

|           |   |   |   |   |   |       |
|-----------|---|---|---|---|---|-------|
| Question: | 1 | 2 | 3 | 4 | 5 | Total |
| Points:   | 1 | 1 | 1 | 1 | 1 | 5     |
| Score:    |   |   |   |   |   |       |

**Short-Answer Questions:**

|           |   |   |    |   |       |
|-----------|---|---|----|---|-------|
| Question: | 6 | 7 | 8  | 9 | Total |
| Points:   | 5 | 8 | 11 | 6 | 30    |
| Score:    |   |   |    |   |       |

# 1 Multiple Choice Questions

Please fill in the bubble(s) **on the exam below** corresponding to your answer. You do not need to submit any additional work for these questions.

1. Given a probability space  $(\Omega, \mathcal{F}, \mathbb{P})$  and three events  $A, B, C \in \mathcal{F}$ , which of the following correctly computes  $\mathbb{P}(A_1 \cup A_2 \cup A_3)$ ? [1pts.]

- $\mathbb{P}(A_1) + \mathbb{P}(A_2) + \mathbb{P}(A_3)$
- $1 - \mathbb{P}(A_1^c) \cdot \mathbb{P}(A_2^c | A_1^c) \cdot \mathbb{P}(A_3^c | A_1^c \cap A_2^c)$
- $\mathbb{P}(A_1) \cdot \mathbb{P}(A_2) \cdot \mathbb{P}(A_3)$
- $1 - \mathbb{P}(A_1) - \mathbb{P}(A_2) - \mathbb{P}(A_3)$
- None of the other answer choices

2. Consider a random variable  $X$  with p.m.f. given by [1pts.]

|          |       |       |
|----------|-------|-------|
| $k$      | $-2$  | $1$   |
| $p_X(k)$ | $1/3$ | $2/3$ |

Which of the following is the correct value of  $\mathbb{E}[X]$ ?

- 0
- $1/3$
- $2/3$
- 1
- 2
- None of the above

3. **Fill in the Blanks:** Discrete random variables have state spaces that are \_\_\_\_\_, [1pts.]  
whereas continuous random variables have state spaces that are \_\_\_\_\_.

- finite; infinite
- countable; uncountable
- at most countable; uncountable
- uncountable; at most countable
- uncountable; countable
- None of the above.

4. In a bag of 100 marbles, 40 are blue and the remaining 60 are gold. Yaz draws marbles one by one at random, replacing the marble each time. If  $X$  denotes the number of marbles (including the final marble) Yaz has to draw before she observes her 3rd blue marble, which of the following accurately describes the distribution of  $X$ ? [1pts.]

- Bern(40)
- Bern(0.4)
- Bin(3, 0.4)
- NegBin(40, 0.4)
- NegBin(3, 0.4)
- HyperGeom(40, 100, 3)
- Poisson(0.4)
- None of the above.

5. Which of the following statements is true in general? [1pts.]

- Pairwise independence implies mutual independence.
- Two mutually dependent events can be conditionally independent
- There are  $2^n$  computations needed to establish the mutual independence of  $n$  events
- Pairwise independence is a stronger condition than mutual independence.
- All of the above answer choices are **false**.

## 2 Short Answer Questions

*Please mark your final answers in the spaces provided below each question. Be sure to show all of your work!*

6. Consider a probability space  $(\Omega, \mathcal{F}, \mathbb{P})$  and suppose  $A$  and  $B$  are two events. [5pts.]  
Prove the identity

$$\mathbb{P}(A \setminus B) = \mathbb{P}(A) \cdot \mathbb{P}(B^c | A)$$

7. The management of *GauchosStay* apartments is quite lazy, and has allowed an ant infestation to manifest. There is a 10% chance that a randomly selected unit will have an infestation problem, independently of all other units. The exterminator goes from unit to unit, but is forgetful and could visit the same unit twice. **For this problem, there is no need to simplify your answers.**

(a) What is the probability that the exterminator observes exactly 3 infested units among the first 7 units they examine? [2pts.]

(b) What is the probability that the 12<sup>th</sup> unit the exterminator examines is the third infested unit they observed? [2pts.]

(c) What is the expected number of units the exterminator must visit before observing their second infested unit? [2pts.]

(d) Now, suppose that there are 100 units in *GauchosStay* and 10 of them are infested. Additionally, suppose that the exterminator now takes care to not examine the same apartment twice. What is the probability that the exterminator observes exactly 3 infested units in a sample of 6 units? [2pts.]

8. Let  $X$  be a continuous random variable with probability density function (p.d.f.) given by

$$f_X(x) = \begin{cases} \frac{2}{9} \cdot x & \text{if } 0 \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Verify that  $f_X(x)$  is a valid probability density function.

[3pts.]

- (b) Compute

$$\mathbb{E} \left[ \frac{1}{(1 + X^2)} \right]$$

[4pts.]

Show all of your steps, including any integration you perform!

- (c) Find  $F_X(x)$ , the cumulative distribution function (c.d.f.) of  $X$ . Be sure to consider all cases! [4pts.]



9. In a drawer, you have 2 red socks, 2 white socks, and 2 green socks. You randomly draw a sample of 4 socks, without replacement; let  $X$  denote the number of matching pairs in your sample (by matching, we mean in color).

(a) What is the state space  $S_X$  of  $X$ ?

[2pts.]

(b) Find the probability mass function of  $X$ .

[4pts.]

(c) Compute  $\mathbb{E}[X]$ .

[1 (bonus)]

*You may use this page for scratch work, if necessary.*

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