Total Points: 10

PSTAT 120A / **QUIZ 1 SOLUTIONS** / Summer 2022 Instructor: **Ethan Marzban**

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1. (1 point) **Multiple Choice.** Which of the following is the correct method needed to evaluate

$$\int \frac{1}{\sqrt{2 - x^2}} \, \mathrm{d}x$$

   - ◯ $u-$substitution
   - ◯ integration by parts
   - √ **inverse trigonometric substitution**
   - ◯ partial fraction decomposition
   - ◯ None of the above.

2. (1 point) **Multiple Choice.** Which of the following is true about the sum

$$\sum_{k=1}^{\infty} \frac{5 \cdot 6^k}{7^k}$$

   - √ **It is convergent**
   - ◯ It diverges to ∞
   - ◯ It diverges to $-\infty$
   - ◯ None of the above.

3. **Short Answer.**

   (a) (4 points) Evaluate $\int \frac{x}{x^2 + 2x + 1} \, \mathrm{d}x$. Show all of your steps and workings!

Solution: We integrate using a $u-$substitution. Let $u = x^2 + 2x + 1$ so that $\mathrm{d}u = 2x + 2 \, \mathrm{d}x$. Then the integral can be written

$$\begin{aligned} \int \frac{x}{x^2 + 2x + 1} \, \mathrm{d}x &= \int \frac{x + 1}{x^2 + 2x + 1} \, \mathrm{d}x - \int \frac{1}{x^2 + 2x + 1} \, \mathrm{d}x \\ &= \int \frac{\frac{1}{2} \, \mathrm{d}u}{u} - \int \frac{1}{(x+1)^2} \, \mathrm{d}x = \frac{1}{2} \ln |u| + \frac{1}{u} + C \\ &= \frac{1}{2} \ln |x^2 + 2x + 1| + \frac{1}{x+1} + C = \ln |x + 1| + \frac{1}{x+1} + C \end{aligned}$$

(b) (4 points) Evaluate $\sum_{k=1}^{\infty} \frac{[\ln(\pi)]^k}{k!}$ Show all of your steps and workings!

Solution: Recall the MacLaurin Series Expansion of e^x:

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$$

Therefore,

$$\sum_{k=0}^{\infty} \frac{[\ln(\pi)]^k}{k!} = e^{\ln(\pi)} = \pi$$

and so

$$\sum_{k=1}^{\infty} \frac{[\ln(\pi)]^k}{k!} = \sum_{k=0}^{\infty} \frac{[\ln(\pi)]^k}{k!} - \frac{[\ln(\pi)]^0}{0!} = \pi - 1$$