Total Points: 10

PSTAT 120A / **QUIZ 2 SOLUTIONS** / Summer 2022 Instructor: **Ethan Marzban**

1. (1 point) **True or False:** Mutual independence implies conditional independence.
   - ◯ True
   - √ **False**

2. (1 point) **Multiple Choice:** If a computer password consists of 10 digits in the following order: 3 digits followed by 7 characters (i.e. alphabet letters), how many possible passwords can be constructed? **Note:** Everyone was awarded 100% on this problem; additionally, the correct answer actually was not one of the listed choices in Gradescope [the answers below are corrected]
   - ◯ $(10)_3 \cdot (26)_7$
   - √ $(10)^3 \cdot (26)^7$
   - ◯ $\binom{10}{3} \cdot \binom{26}{7}$
   - ◯ None of the other answer choices.

3. Suppose that in a specific quarter, PSTAT 120A contains 100 students. 50 of these students are enrolled in the statistics major, and 30 are enrolled in only the mathematics major (but not the statistics major). Additionally, 10 of these students are double majors between statistics and mathematics.

   (a) (3 points) What is the probability that a randomly selected PSTAT 120A student is enrolled in only the statistics major? **Be explicit about your notation!**

   > **Solution:** Let S denote the event "student is enrolled in the statistics major" and M denote the event "student is enrolled in the mathematics major." Then, we are told that
   > $$\mathbb{P}(S) = 0.5; \quad \mathbb{P}(M \setminus S) = 0.3; \quad \mathbb{P}(M \cap S) = 0.1$$
   > Thus, we have that $\mathbb{P}(S \setminus M) = \mathbb{P}(S) - \mathbb{P}(S \cap M) = 0.5 - 0.1 = 0.4$

   (b) (3 points) What is the probability that a randomly selected PSTAT 120A student is enrolled in neither the statistics major nor the mathematics major?

   > **Solution:** We seek $\mathbb{P}(S^\complement \cap M^\complement)$. By DeMorgan's Law, this is computable as $1 - \mathbb{P}(S \cup M) = 1 - \mathbb{P}(S) + \mathbb{P}(M \setminus S) = 1 - (0.5 + 0.3) = 0.2$

   (c) (2 points) Given that John is in the statistics major, what is the probability that he is also in the mathematics major?

Solution: We seek $\mathbb{P}(M \mid S)$, which can be computed using the Multiplication Rule:

$$\mathbb{P}(M \mid S) = \frac{\mathbb{P}(M \cap S)}{\mathbb{P}(S)} = \frac{0.1}{0.5} = \frac{1}{5} = 20\%$$