

**PSTAT 120A, Summer 2022: Practice Problems 3***Week 2**Conceptual Review*

- (a) What does expected value measure?
- (b) What does variance measure?

*Problem 1: Linearity of Expectation*

Consider a probability space  $(\Omega, \mathcal{F}, \mathbb{P})$  and a random variable  $X$  with expectation  $\mu := \mathbb{E}[X]$ .

- (a) Prove that  $\mathbb{E}[aX + b] = a\mu + b$ .
- (b) Prove that  $\mathbb{E}[g(X) + h(X)] = \mathbb{E}[g(X)] + \mathbb{E}[h(X)]$

*Problem 2: Verifying P.M.F's*

Let  $X$  be a random variable with p.m.f. given by

$$p_X(k) = \begin{cases} c & \text{if } k = 0 \\ \left(\frac{1}{3}\right)^k & \text{if } k = 1, 2, \dots \\ 0 & \text{otherwise} \end{cases}$$

- (a) Find the value of  $c$  that ensures  $p_X(k)$  is a valid probability measure.
- (b) Compute the probability that  $X$  is even. (Recall that 0 is even.)
- (c) Compute  $\mathbb{E}[X]$
- (d) It can be shown that  $\mathbb{E}[X^2] = 3/2$ . Compute  $\text{Var}(X)$

*Problem 3: Proofreader*

In a given book, each page contains a typo with probability 10% independently of all other pages. An editor begins examining the book page-by-page.

- (a) What is the probability that among the first 10 pages examined the editor will find exactly 3 typos?
- (b) What is the probability that among the first 10 pages examined the editor will find at most 3 typos?
- (c) What is the probability that the 4<sup>th</sup> page the editor examines is the first page to contain a typo?
- (d) What is the expected number of pages, including the final page, that the editor will need to examine before observing the first typo?

**Extra Problems**

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*Problem 4: Variance of the Geometric Distribution*

Let  $X \sim \text{Geom}(p)$ .

- (a) Compute  $\mathbb{E}[X(X - 1)]$ .
- (b) Using your answer to part (a), find  $\mathbb{E}[X^2]$ .
- (c) Using your answer to part (b), show that  $\text{Var}(X) = 1/p^2$ .

*Hint: Try differentiating the geometric series repeatedly*